

Gross Stability of a Liner-on-Plasma System Near the Stagnation Point

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Abstract. Two issues related to the liner stability are considered: a) the presence of a thin “magnetic cushion” between the liner and the hot dense plasma at the deceleration phase near stagnation; b) an account for volumetric density perturbations as a seed for the instability growth at the acceleration phase.

Keywords: Liner stability; surface perturbations; volumetric perturbations.

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I. INTRODUCTION

In this paper we consider some stability properties of cylindrical liners driven by the axial current and compressing a dense plasma (see Refs. 1, 2 and references therein for further details). The most dangerous mode is the axisymmetric, $m=0$, “sausage” mode that develops near the outer surface during the acceleration phase and is then fed-through to the inner surface, to seed the instability of the plasma-liner interface near stagnation. In order to reach a reasonable energy efficiency of the system, it is beneficial to keep the magnetic field in the compressed plasma low, so that the magnetic pressure would be very small compared to the plasma pressure, and the liner energy would be converted mostly to the plasma energy (with that, the field can still be sufficient to suppress radial heat losses).

However, near the liner-plasma interface, the plasma magnetic field is piled up by the plasma “cooling flow” directed towards the liner, and a thin layer with a high field, where the ratio β of the magnetic pressure to the plasma pressure is ~ 1 , is formed (e.g., [2,3]). As the compressed magnetic field is directed predominantly along the axis, this “magnetic cushion” would have a stabilizing effect on the $m=0$ perturbations: these perturbations cause bending of the field lines near the wall and thereby create a restoring force. The corresponding stabilizing effect was considered in Ref. [4]. Here we provide some quantitative details regarding its magnitude (Sec. II).

In Sec. III, we consider another aspect of the instability: an effect of volumetric density perturbations in the seeding the instability at the acceleration phase. We focus on the $m=0$ mode, as this mode is best studied experimentally [5, 6]. We find a convenient universal way of comparing the role of surface perturbations and volumetric perturbations.

The liner is considered as an incompressible, inviscid fluid. The liner thickness h is assumed to be significantly less than the liner radius. As the most dangerous perturbations are those with the scale-length smaller than or comparable to the liner thickness, one can safely use a planar model, similar to that considered by Harris [7].

II. STABILIZING EFFECT OF THE MAGNETIC CUSHION

General equations characterizing the stabilization by the magnetic cushion for the slowing down liner (near stagnation) have been derived in Ref. [4]. The dimensionless dispersion relation reads as:

$$\Gamma^4 + \frac{2\Gamma^2 q^2}{1+\beta} (\tanh q \frac{h_M}{h}) (\coth q) + q \left[\frac{2q^2}{1+\beta} (\tanh q \frac{h_M}{h}) - q \right] = 0, \quad (1)$$

where h and h_m are the liner thickness and the thickness of the magnetic cushion, respectively, $q \equiv |k_z| h$, and Γ is a growth-rate normalized to $\sqrt{g/h}$. This dispersion relation shows that, as a function of q , the growth rate first increases, reaches a maximum, and then drops to zero [4].

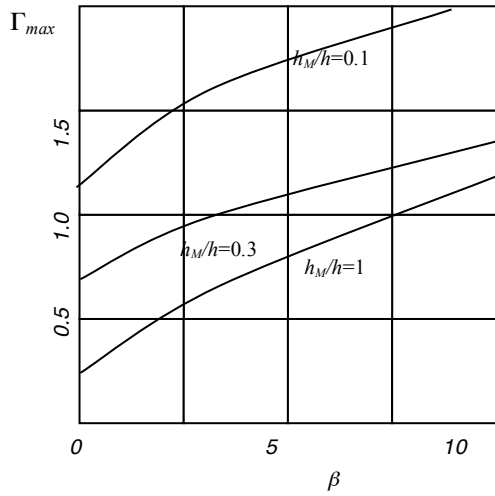


Fig. 1. The effect of the magnetic cushion on the liner stability near the stagnation point. Lower betas and larger thicknesses lead to a significant increase of the maximum growth rate. The parameter b is the ratio of the plasma to magnetic field pressure in the “cushion.” The growth rate is measured in the units of $\sqrt{g/h}$.

In Fig. 1 the impact of the magnetic cushion on the instability is characterized by the *maximum* (over q) growth rate as a function of the plasma beta in the magnetic cushion, for several relative thicknesses of the cushion, h_m/h . The smallest value, $h_m/h=0.1$, corresponds to a virtual absence of the cushion. However, even for $h_m/h=0.3$, and $\beta=2$, the maximum growth rate drops by a factor ~ 2 compared to the absence of the cushion and higher β .

Not affected by the magnetic cushion are the flute perturbations of the inner surface (the ones that have no z dependence and, therefore, do not cause bending of the axial magnetic field lines). On the other hand, it is believed that their non-linear saturation would occur at lower amplitudes than for the $m=0$ perturbations.

III. EFFECT OF THE VOLUMETRIC DENSITY PERTURBATIONS

We focus on the $m=0$ mode during the acceleration phase. The role of volumetric perturbations in seeding the instability has been discussed in conjunction with numerical simulations of experiments [5, 6]. Here we provide a simple analytical model, for the geometry shown in Fig. 2.

The dynamics of an incompressible fluid with small initial density perturbations $\delta\rho \ll \rho$ is described by the equations:

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -\nabla \delta p + g \delta \rho, \quad (2)$$

$$\nabla \cdot \xi = 0, \quad (3)$$

where ξ is an infinitesimal displacement of a fluid element with respect to its initial position. The linearized continuity equation shows that $\delta \rho$ does not depend on time. The density non-uniformity may be caused, for example, by the composition non-uniformity.

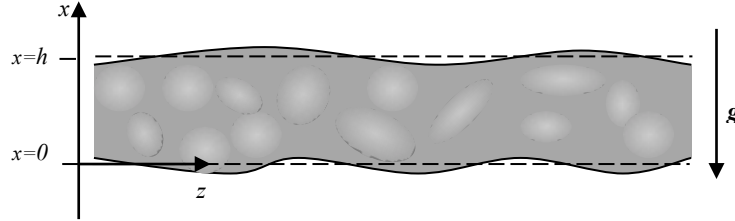


Fig. 2. The geometry of the system. The drive magnetic field is applied in the lower half-space ($x < 0$) and is directed towards the viewer. The effective gravity acceleration in the frame co-moving with the liner is directed downward. The axis z corresponds to the liner axis in the cylindrical geometry. Shown are volumetric density perturbations and the surface waviness that seed the instability.

The vector g is directed downward. The boundary conditions at the interfaces $x=0$ and $x=h$ are (Cf. Refs. [7, 8]):

$$\delta p - \rho_0 |g| \xi_x = 0 \text{ at } x=0, h. \quad (4)$$

Applying Eq. (3) to Eq. (2), one finds that $-\nabla^2 \delta p - |g| \delta \rho' = 0$, where prime designates the x -derivative. This equation for a single axial mode can be rewritten as

$$\delta p'' - k_z^2 \delta p = -|g| \delta \rho' \quad (5)$$

An analog of the $m=0$ mode in the planar geometry is the mode where all the quantities depend on time and two of the spatial variables, x and z . For the linear perturbations, one can look for the perturbations whose z dependence is $\exp(ik_z z)$.

By solving Eq. (5) with the boundary conditions (4), one can express the pressure perturbation in terms of the surface displacements, $\xi_l \equiv \xi_x|_{x=0}$ and $\xi_u \equiv \xi_x|_{x=h}$, where the subscripts “ l ” and “ u ” refer to the lower and upper surfaces (Fig. 1). The result reads as:

$$\delta p = \rho_0 |g| \left[\frac{e^{kx} - e^{-kx}}{e^{kh} - e^{-kh}} (\xi_u - F(h)) - \frac{e^{k(x-h)} - e^{-k(x-h)x}}{e^{kh} - e^{-kh}} \xi_l + F(x) \right], \quad (6)$$

$$F(x) = -\frac{1}{2k\rho_0} \left[e^{kx} \int_0^x \delta \rho'(x_1) e^{-kx_1} dx_1 - e^{-kx} \int_0^x \delta \rho'(x_1) e^{kx_1} dx_1 \right], \quad (7)$$

where x_l is an integration variable. Note that $F(0) = F'(0) = 0$. Note also that here and below we use a shorthand $k = k_z$.

Substituting Eq. (6) into Eq. (2) and applying the resulting equation at the points $x=0, h$, one can find the dynamical system that describes the evolution of the two surfaces. Before presenting this system, we introduce new variables characterizing the surface perturbations:

$$\tilde{\xi}_u = \xi_u + \frac{1}{\sinh(kh)} \int_0^h \frac{\delta \rho(x_1)}{\rho_0} \sinh(kx_1) dx_1; \quad \tilde{\xi}_l = \xi_l + \frac{1}{\sinh(kh)} \int_0^h \frac{\delta \rho(x_1)}{\rho_0} \sinh(kh - kx_1) dx_1. \quad (8)$$

The dynamic equations then acquire the form:

$$\frac{\partial^2 \tilde{\xi}_u}{\partial t^2} = |g|k \left[-\frac{\tilde{\xi}_u}{\tanh(kh)} + \frac{\tilde{\xi}_l}{\sinh(kh)} \right]; \quad \frac{\partial^2 \tilde{\xi}_l}{\partial t^2} = |g|k \left[-\frac{\tilde{\xi}_u}{\sinh(kh)} + \frac{\tilde{\xi}_l}{\tanh(kh)} \right]. \quad (9)$$

To get this set of equations, one has to perform integrations by part in Eq. (7).

What is remarkable in the set (9) is that it is exactly the same as that for the system without density non-uniformities [7]. All the information about the effect of non-uniformities is encapsulated in Eqs. (8) relating the real displacements, ξ_l and ξ_u to the auxiliary ones, $\tilde{\xi}_l$ and $\tilde{\xi}_u$. If there are no initial surface perturbations, the initial conditions for $\tilde{\xi}_l$ and $\tilde{\xi}_u$ would be Eqs. (8) with $\tilde{\xi}_l(t=0) = \tilde{\xi}_u(t=0) = 0$. All the issues of the feed-through would then be treated exactly in the same way as for the “standard” (without density perturbations) system. Note a detailed discussion of the feed-through issues for the “standard” system presented in Ref. [9].

As an example, one can consider the initial value problem for the instability of a semi-infinite slab ($kh \gg 1$). In this case, the general solution for the instability of the interface with a finite initial displacement $\xi_l(t=0) = \xi_0$ and zero initial velocity $\dot{\xi}_l(t=0) = 0$ read as:

$$\xi_l = \xi_0 \cosh(\Gamma t) + [\cosh(\Gamma t) - 1] \int_0^\infty \delta\rho(x) \exp(-kx) dx, \quad (10)$$

where $\Gamma = \sqrt{k|g|}$. It should be remembered that both ξ_l and $\delta\rho$ are actually the k -components of the Fourier transform of the initial perturbations.

In this report, only linear stage of the instability growth was treated. This is why the effect of the density perturbations enters the problem via the initial density distribution and does not contain the time dependence. This dependence appears in the second order, via the terms like . We leave an analysis of the corresponding constraints as well as a detailed comparison with the existing experimental results for future work.

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